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Journal of
APPLIED MATHEMATICS AND MECHANICS

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Journal of Applied Mathematics and Mechanics 69 (2005) 305–314

THE SLIDING OF VISCOELASTIC BODIES WHEN THERE IS ADHESION†

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(*Received* 16 *December* 2003)

The plane contact problem of the sliding without friction of a rigid cylinder over a viscoelastic half-space when there is adhesion is solved, neglecting the inertial properties of the half-space. The distribution of the contact pressure, the size and position of the contact area, and the deformation force of resistance to motion of the cylinder are investigated as a function of the adhesion properties of the surfaces, the mechanical characteristics of the half-space and the sliding velocity of the cylinder.

In order to investigate the contact interaction of viscoelastic bodies during sliding, the problem of the sliding of a rigid cylinder over a viscoelastic half-space was solved in the plane formulation in [1, 2] by a method based on reducing it to a Riemann–Hilbert problem. A similar method is used below to investigate the role of adhesion during the sliding of a cylinder over a viscoelastic half-space. Earlier, effects of adhesion during the contact of viscoelastic bodies were studied in [3] in the case when the bodies were moving towards or away each other in the normal direction.

1. FORMULATION OF THE PROBLEM

The plane contact problem is considered for a viscoelastic half-space, the properties of which are described by the equations

$$\begin{aligned}
\varepsilon_{x^0} + T_\varepsilon \frac{\partial \varepsilon_{x^0}}{\partial t} &= \frac{1-\nu^2}{E}\left(\sigma_{x^0} + T_\sigma \frac{\partial \sigma_{x^0}}{\partial t}\right) - \frac{\nu(1+\nu)}{E}\left(\sigma_{y^0} + T_\sigma \frac{\partial \sigma_{y^0}}{\partial t}\right) \\
\varepsilon_{y^0} + T_\varepsilon \frac{\partial \varepsilon_{y^0}}{\partial t} &= \frac{1-\nu^2}{E}\left(\sigma_{y^0} + T_\sigma \frac{\partial \sigma_{y^0}}{\partial t}\right) - \frac{\nu(1+\nu)}{E}\left(\sigma_{x^0} + T_\sigma \frac{\partial \sigma_{x^0}}{\partial t}\right) \qquad (1.1) \\
\gamma_{x^0y^0} + T_\varepsilon \frac{\partial \gamma_{x^0y^0}}{\partial t} &= \frac{1+\nu}{E}\left(\tau_{x^0y^0} + T_\sigma \frac{\partial \tau_{x^0y^0}}{\partial t}\right)
\end{aligned}$$

where σ_{x^0}, σ_{y^0} and $\tau_{x^0y^0}$ are the stress tensor components, ε_{x^0}, ε_{y^0} and $\gamma_{x^0y^0}$ are the strain tensor components, E is the Young's modulus, ν is Poisson's ratio, T_ε and T_σ characterize the viscous properties of the half-space, (x^0, y^0) is the system of coordinates connected with the half-space and t is the time. Equations (1.1) are a two-dimensional analogue of the Maxwell–Thomson model and correspond to the case of plane strain.

A rigid cylinder of radius R slides along the boundary of a viscoelastic half-space at a velocity w (Fig. 1). It is assumed that there is no friction between the cylinder and the half-space. The inertial properties of the half-space will be ignored.

†*Prikl. Mat. Mekh.* Vol. 69, No. 2, pp. 334–344, 2005.

0021–8928/$—see front matter.
doi: 10.1016/j.jappmathmech.2005.03.017

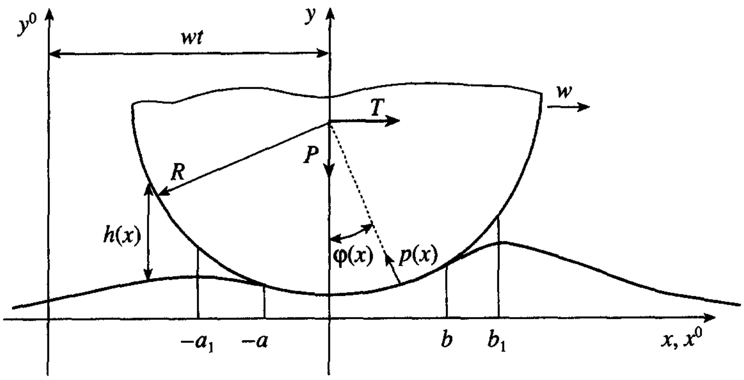

Fig. 1

To describe the adhesion interaction of the surfaces, we will use the model [4] according to which the force of attraction of the surfaces per unit area is approximated by a piecewise-constant function

$$p_a(h) = \begin{cases} p_0, & 0 < h \le h_0 \\ 0, & h > h_0 \end{cases} \tag{1.2}$$

where h is the size of the gap between the interacting surfaces. The surface energy of interaction γ is defined by the relation

$$\gamma = \int_0^\infty p_a(h)dh = p_0 h_0$$

This leads to the condition for the maximum size of the gap h_0, for which the surfaces still experience adhesion attraction

$$h_0 = \gamma / p_0 \tag{1.3}$$

The surface energy γ and the adhesion pressure p_0 are considered to be specified quantities.

We will introduce a moving system of coordinates (x, y) connected with the cylinder: $x = x^0 - wt$, $y = y^0$. Assuming that the shape of the cylinder in the vicinity of the region of interaction with the half-space can be described by the function $f(x) = x^2/(2R)$, we obtain the following boundary conditions for the viscoelastic half-space in the moving system of coordinates (x, y):

the conditions in the contact area

$$\frac{\partial \upsilon}{\partial x} = \frac{x}{R}, \quad \tau_{xy}|_{y=0} = 0, \quad -a < x < b \tag{1.4}$$

where $\upsilon(x)$ is the normal displacement of the boundary of the viscoelastic half-space, the conditions in the region of adhesion interaction

$$\sigma_y|_{y=0} = p_0, \quad \tau_{xy}|_{y=0} = 0, \quad -a_1 \le x \le a, \quad b \le x \le b_1 \tag{1.5}$$

and the condition for there to be no load outside the interaction region

$$\sigma_y|_{y=0} = 0, \quad \tau_{xy}|_{y=0} = 0, \quad x < -a_1, \quad x > b_1 \tag{1.6}$$

In the moving system of coordinates $(x. y)$ the stresses, strains and displacements do not depend explicitly on time. In particular, for normal displacement of the boundary of the half-space we have $\upsilon(x) \equiv \upsilon^0(x + wt, t)$, where $\upsilon^0(x^0, t)$ is the normal displacement in the moving system of coordinates (x^0, y^0). Differentiating this identity with respect to time, we obtain $\partial\upsilon(x^0, t)/\partial t = -w\partial\upsilon(x)/\partial x$. Similar relations hold for the derivatives of all the components of the stresses, displacements and strains.

We will introduce the following notation

$$\varepsilon_x^* = \varepsilon_{x^0} - T_\varepsilon w\frac{\partial\varepsilon_x}{\partial x}, \quad \varepsilon_y^* = \varepsilon_{y^0} - T_\varepsilon w\frac{\partial\varepsilon_y}{\partial x}, \quad \gamma_{xy}^* = \gamma_{x^0y^0} - T_\varepsilon w\frac{\partial\gamma_{xy}}{\partial x}$$

$$\sigma_x^* = \sigma_{x^0} - T_\sigma w\frac{\partial\sigma_x}{\partial x}, \quad \sigma_y^* = \sigma_{y^0} - T_\sigma w\frac{\partial\sigma_y}{\partial x}, \quad \tau_{xy}^* = \tau_{x^0y^0} - T_\sigma w\frac{\partial\tau_{xy}}{\partial x} \tag{1.7}$$

$$\upsilon^* = \upsilon - T_\varepsilon w\frac{\partial\upsilon}{\partial x}$$

Then, Eqs (1.1) take a form identical with that of Hooke's law for an elastic half-space. The functions introduced with an asterisk also satisfy the equations of equilibrium and consistency of the strains for an elastic body.

In order to obtain the boundary conditions for the functions with an asterisk in (1.7), we will use conditions (1.4)–(1.6). From (1.5) and (1.6) it can be seen that the function σ_y when $y = 0$ outside the contact area can be represented using Heaviside's θ-function

$$\sigma_y|_{y=0} = \begin{cases} p_0\theta(x+a_1), & x \le -a \\ p_0\theta(b_1 - x), & x \ge b \end{cases}$$

Taking this into account, we obtain the following boundary conditions at $y = 0$ for the functions with an asterisk

$$\partial\upsilon^*/\partial x = (x - T_\varepsilon w)/R, \quad \tau_{xy}^* = 0, \quad -a < x < b$$

$$\sigma_y^* = p_0\theta(x+a_1) - T_\sigma w p_0\delta(x+a_1), \quad \tau_{xy}^* = 0, \quad x \le -a \tag{1.8}$$

$$\sigma_y^* = p_0\theta(b_1 - x) + T_\sigma w p_0\delta(b_1 - x), \quad \tau_{xy}^* = 0, \quad x \ge b$$

where $\delta(x)$ is delta function.

Thus, the initial problem reduces to solving the problem for an elastic half-space with boundary conditions (1.8). After this, the true stresses, strains and displacements in a viscoelastic half-space can be determined from the solution of differential equations (1.7).

2. METHOD OF SOLUTION

Solution of the problem for an elastic half-space. To solve the problem for an elastic half-space with boundary conditions (1.8), we will use Galin's method [5] and introduce, for $y \le 0$, the function of the complex variable $z = x - iy$

$$W(z) = U - iV = \int_{-\infty}^{+\infty} \sigma_y^* \frac{dt}{t-z} \tag{2.1}$$

Since at the boundary $y = 0$ of the elastic half-space the relation

$$\frac{\pi E}{2(1-\nu^2)}\frac{\partial\upsilon^*}{\partial x} = \int_{-\infty}^{+\infty} \sigma_y^* \frac{dt}{t-x} - \frac{1-2\nu}{2-2\nu}\pi\tau_{xy}^* \tag{2.2}$$

is satisfied [5], taking the limit value of Cauchy-type integral (2.1) when $z \to x - i0$ into account, from conditions (1.8) we obtain the boundary conditions for the function $W(z)$

$$U(x, 0) = \frac{\pi E}{2(1-\nu^2)R}(x - T_\varepsilon w), \quad -a < x < b$$

$$V(x, 0) = \begin{cases} \pi p_0\theta(x+a_1) - \pi T_\sigma w p_0\delta(x+a_1), & x \le -a \\ \pi p_0\theta(b_1 - x) + \pi T_\sigma w p_0\delta(b_1 - x), & x \ge b \end{cases} \tag{2.3}$$

The problem of determining the function $W(z)$, which is analytical when $y \le 0$, for the prescribed boundary conditions (2.3) at $y = 0$ is a special case of the Riemann–Hilbert problem. The solution of this problem, which behaves like P/z as $z \to \infty$, has the form [5]

$$W(z) = \frac{1}{\pi\sqrt{Z(z)}}\left[P + \int_{-a}^{b} U(x,0)\sqrt{-Z(x)}\frac{dx}{x-z} - \right.$$

$$\left. - \int_{-\infty}^{-a} V(x,0)\sqrt{-Z(x)}\frac{dx}{x-z} + \int_{b}^{\infty} V(x,0)\sqrt{Z(x)}\frac{dx}{x-z}\right] \quad (2.4)$$

Here

$$Z(z) = (z+a)(z-b)$$

P is the normal external force acting on the cylinder (Fig. 1).

Knowing the function W, it is possible to determine the stress σ_y^* and displacements υ^* at the boundary $y = 0$ from the relations [5]

$$U(x,0) = E^*\frac{\partial \upsilon^*}{\partial x}, \quad V(x,0) = \pi\sigma_y^*\big|_{y=0}, \quad E^* = \frac{\pi E}{2(1-\nu^2)} \quad (2.5)$$

Taking these relations into account, after taking the integrals in (2.4), we obtain the following expressions for stress σ_y^* and displacements υ^*

$$\sigma_y^*\big|_{y=0} = -p^*(x) = [-F(x) + G(x)]/\pi, \quad -a < x < b$$

$$\frac{\partial \upsilon^*}{\partial x} = \frac{x - T_\varepsilon w}{R} + \begin{cases} [H(x) - F(x)]/E^*, & -a_1 < x < -a \\ [F(x) - H(x)]/E^*, & b < x < b_1 \end{cases} \quad (2.6)$$

where

$$F(x) = \frac{1}{\sqrt{|Z(x)|}}\left\{\frac{E^*}{R}\left[\frac{1}{8}(a+b)^2 + \frac{1}{2}(x - T_\varepsilon w)(b - a - 2x)\right] + \right.$$

$$+ P + \frac{1}{2}p_0 \ln\frac{2a_1 + b - a + 2\sqrt{Z(-a_1)}}{2b_1 - b + a + 2\sqrt{Z(b_1)}} +$$

$$\left. + p_0[\sqrt{Z(-a_1)} + \sqrt{Z(b_1)}] + T_\sigma w p_0\left[\frac{\sqrt{Z(b_1)}}{b_1 - x} - \frac{\sqrt{Z(-a_1)}}{a_1 + x}\right]\right\}$$

$$G(x) = p_0\left\{\pi + \operatorname{arctg}\frac{(a_1 - a)(b - x) - (a_1 + b)(a + x)}{2\sqrt{Z(-x)Z(-a_1)}} - \right. \quad (2.7)$$

$$\left. - \operatorname{arctg}\frac{(b_1 + a)(b - x) - (b_1 - b)(a + x)}{2\sqrt{Z(-x)Z(b_1)}}\right\}$$

$$H(x) = p_0\left\{-\operatorname{arcth}\frac{(a_1 - a)(x - b) + (a_1 + b)(x + a)}{2\sqrt{Z(x)Z(-a_1)}} + \right.$$

$$\left. + \operatorname{arcth}\frac{(b_1 + a)(x - b) + (b_1 - b)(x + a)}{2\sqrt{Z(x)Z(b_1)}}\right\}$$

Determination of the true stresses and displacements at the boundary of a viscoelastic half-space. The stresses and displacements at the boundary of a viscoelastic half-space are determined by solving differential equations (1.7). The conditions of continuity for these stresses and displacements are used as the boundary conditions. Taking the condition $p(-a) = -p_0$ into account, where $p(x) = -\sigma_y|_{y=0}$ is the contact pressure, we will obtain, in the contact area $-a < x < b$, the expression

$$p(x) = -\frac{1}{T_\sigma w}\int_{-a}^{x} p^*(t)e^{(x-t)/(T_\sigma w)}dt - p_0 e^{(x+a)/(T_\sigma w)} \tag{2.8}$$

Taking into account the condition $\partial v/\partial x = -a/R$ at $x = -a$, we obtain the following expression for the displacement when $-a_1 < x < -a$:

$$\frac{\partial v(x)}{\partial x} = \frac{1}{T_\varepsilon w}\int_{x}^{-a} \frac{\partial v^*(t)}{\partial t}e^{(x-t)/(T_\varepsilon w)}dt - \frac{a}{R}e^{(x+a)/(T_\varepsilon w)} \tag{2.9}$$

In order to obtain an expression for the displacements in the region $b < x < b_1$, we will use the condition $\partial v/\partial x = b/R$ when $x = b$

$$\frac{\partial v(x)}{\partial x} = -\frac{1}{T_\varepsilon w}\int_{b}^{x} \frac{\partial v^*(t)}{\partial t}e^{(x-t)(/T_\varepsilon w)}dt + \frac{b}{R}e^{(x-b)/(T_\varepsilon w)} \tag{2.10}$$

The relations (2.8)–(2.10), in which the functions $p^*(x)$ and $\partial v^*(x)/\partial x$ are specified by expressions (2.6), define the normal pressure and the normal displacements at the boundary of the viscoelastic half-space.

Determination of the boundaries of the contact areas and adhesion interaction. Relations (2.8)–(2.10), which define the stresses and displacements at the boundary of the viscoelastic half-space, contain the unknown quantities, a, b, a_1 and b_1 – the coordinates of the boundaries of the area of contact of the cylinder with the half-space ($-a$ and b) and the external boundaries of the region of adhesion interaction of the surfaces ($-a_1$ and b_1). To determine these four unknown quantities, four conditions are needed.

The first condition stems from the remaining unused condition of continuity of the contact stress at $x = b$. From expression (2.8), taking into account condition $p(b) = -p_0$, we obtain

$$-\frac{1}{T_\sigma w}\int_{-a}^{b} p^*(x,0)e^{-x/(T_\sigma w)}dx + p_0(1 - e^{(a+b)/(T_\sigma w)}) = 0 \tag{2.11}$$

The second condition is obtained from the condition for the strains to attenuate at the boundary of the half-space at $x \to \infty$. The application of this condition to relation (2.10) gives

$$\frac{1}{T_\varepsilon w}\int_{b}^{+\infty} \frac{\partial v^*(x)}{\partial x}e^{-x/(T_\varepsilon w)}dw - \frac{b}{R}e^{-b/(T_\varepsilon w)} = 0 \tag{2.12}$$

The third and fourth conditions follow from the fact that the size of the gap between the surfaces of the cylinder and the half-space at the points $x = -a_1$ and $x = b_1$ should be equal to the maximum distance h_0 at which the surfaces still experience adhesion attraction (see (1.2)). Equating the gap size at the point $x = -a_1$ to the quantity h_0 defined by relation (1.3), we obtain

$$f(-a_1) - f(-a) - v(-a_1) + v(-a) = \gamma/p_0$$

from which it follows that

$$\int_{-a_1}^{-a} \frac{\partial v(x)}{\partial x}dx = \frac{\gamma}{p_0} - \frac{a_1^2 - a^2}{2R}$$

Substituting into this relation the expression for the derivative of the normal displacement of the boundary of the half-space (2.9) when $-a_1 < x < -a$, and changing the limits of integration, we obtain the condition

$$\int_{-a_1}^{-a} \frac{\partial v^*(x)}{\partial x}\left(1 - e^{-\frac{a_1+x}{T_\varepsilon w}}\right)dx - \frac{aT_\varepsilon w}{R}\left(1 - e^{-\frac{a_1-a}{T_\varepsilon w}}\right) = \frac{\gamma}{p_0} - \frac{a_1^2 - a^2}{2R} \tag{2.13}$$

In a similar way, equating the gap size at the point $x = b_1$ to the magnitude of h_0, we find the fourth condition

$$\int_{b}^{b_1} \frac{\partial v^*(x)}{\partial x}\left(e^{\frac{b_1-x}{T_\varepsilon w}} - 1\right)dx - \frac{bT_\varepsilon w}{R}\left(e^{\frac{b_1-b}{T_\varepsilon w}} - 1\right) = \frac{\gamma}{p_0} - \frac{b_1^2 - b^2}{2R} \tag{2.14}$$

The system of four equations (2.11)–(2.14) obtained for determining the four unknowns a, b, a_1 and b_1 was solved numerically by Newton's method.

3. THE CASE OF AN ELASTIC HALF-SPACE

We will consider the case of an elastic half-space interacting with a rigid cylinder when there is adhesion. In this case we obtain the contact problem for an elastic half-space with boundary conditions (1.5)–(1.6). Solving this problem by a method similar to that set out in Section 2, we find the following expressions for the contact pressure when $-a \le x \le a$ (the solution is symmetrical about the Oy axis, i.e. $b = a$ and $b_1 = a_1$)

$$\begin{aligned} p(x) &= \frac{E^*}{\pi R}\sqrt{a^2 - x^2} + \frac{p_0}{\pi}[\xi_+(x) - \xi_-(x) - \pi] \\ \xi_\pm(x) &= \operatorname{arctg}\frac{a_1 x \pm a^2}{\sqrt{a^2 - x^2}\sqrt{a_1^2 - a^2}} \end{aligned} \tag{3.1}$$

and the elastic displacement of the boundary of the half-space when $a \le x \le a_1$

$$\begin{aligned} \frac{dv}{dx} &= \frac{x}{R} - \frac{\sqrt{x^2 - a^2}}{R} + \frac{p_0}{E^*}[\eta_+(x) - \eta_-(x)] \\ \eta_\pm(x) &= \operatorname{arcth}\frac{a_1 x \mp a^2}{\sqrt{x^2 - a^2}\sqrt{a_1^2 - a^2}} \end{aligned} \tag{3.2}$$

Furthermore, we obtain an expression for the load applied to the cylinder.

$$P = E^* a^2/(2R) - 2p_0\sqrt{a_1^2 - a^2} \tag{3.3}$$

and the condition

$$a_1\sqrt{a_1^2 - a^2} + \left(a^2 - \frac{4p_0}{E^*}\sqrt{a_1^2 - a^2}\right)\ln\frac{a_1 - \sqrt{a_1^2 - a^2}}{a} = \frac{2R\gamma}{p_0} \tag{3.4}$$

which was obtained by equating the size of the gap at the point $x = a_1$ to the magnitude of h_0, defined by relation (1.3). Relations (3.3) and (3.4) are used for the numerical determination of the coordinates a and a_1 of the boundaries of contact area and adhesion interaction.

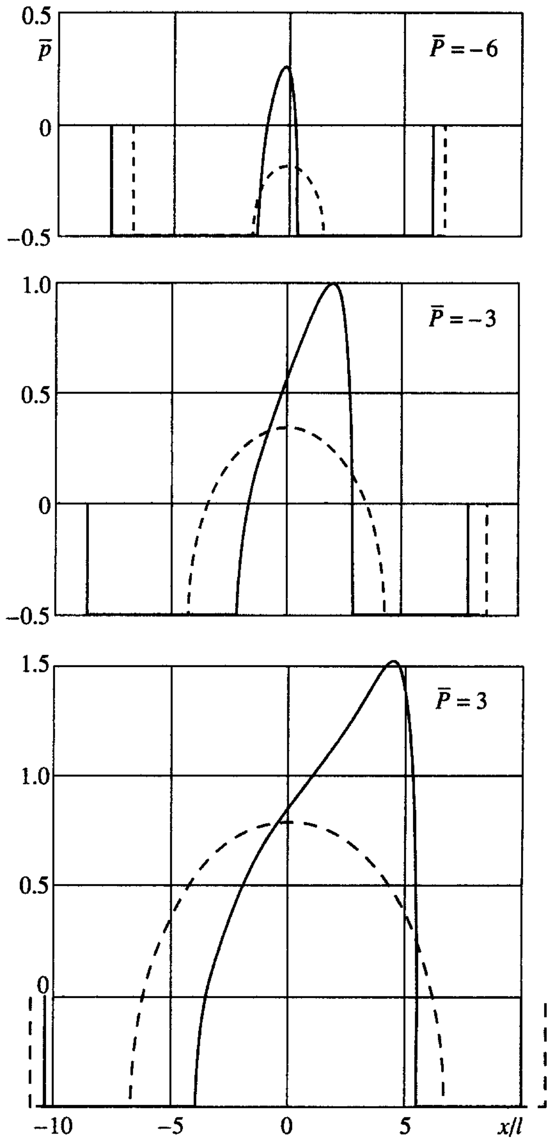

Fig. 2

4. RESULTS OF CALCULATIONS

In the course of a numerical solution of the problem, we investigated how the solution depends on the dimensionless parameters characterizing the viscous properties of the half-space $\delta = T_\varepsilon/T_\sigma$, the adhesion of the surfaces $\lambda = p_0[9R\pi/(8E^{*2}\gamma)]^{1/3}$, the sliding velocity of the cylinder, the delay of the material of the half-space $\kappa = l/2wT_\varepsilon)$, and also the magnitude of the normal load $\bar{P} = \lambda P(lp_0)$, where $l = [R^2\gamma/(9\pi E^*)]^{1/3}$.

Figure 2 shows the distributions of the dimensionless contact pressure $\bar{p} = \lambda p/p_0$ with respect to the dimensionless coordinate x/l for $\delta = 1$, $\lambda = 0.5$ and various values of the load. The dashed lines indicate the pressure distribution with the same parameters for an elastic half-space (relations (3.1), (3.3) and (3.4)); these results are identical with the solution obtained for a viscoelastic half-space with $\delta = 1$. It can be seen that allowance for the viscous properties of the half-space leads to a reduction in the contact area and an increase in the maximum contact pressure. Furthermore, the distribution of the contact pressure becomes asymmetrical. For positive values of the load $\bar{P}$ (the external force presses the cylinder against the half-space) the contact area is shifted in the direction of motion of the cylinder. At fairly high negative loads (the external force separates the cylinder and the half-space) the contact area is shifted in the opposite direction to the motion of the cylinder. There is a similar shift in the region of

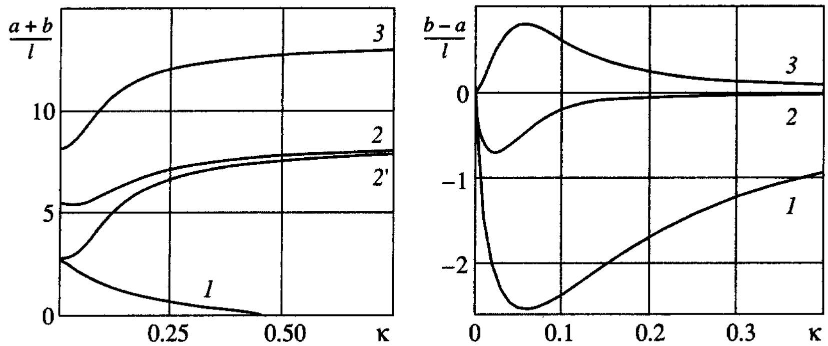

Fig. 3

adhesion interaction of the surfaces, in which the dimensionless pressure at the boundary of the viscoelastic space is constant and equal to $p = -\lambda$.

The dependence of the dimensionless width of the contact area $(a + b)/l$ and its shift with respect to the axis of the symmetry of the cylinder $(b - a)/l$ on the parameter κ is presented in Fig. 3 for $\lambda = 0.5$, $\delta = 3$ and different loads: $\bar{P} = -6.3$ (curve *1*), $\bar{P} = -3$ (curve *2*) and $\bar{P} = 3$ (curve *3*). The results of the calculations enable us to conclude that the width of the contact area is restricted by the limit values it has when $\kappa \to 0$ and $\kappa \to \infty$. The case when $\kappa \to \infty$, i.e. $w \to 0$, corresponds to the solution of the problem of the interaction of a cylinder with an elastic half-space characterized by the modulus E. In the case when $\kappa \to 0$, the viscoelastic half-space behaves like an elastic body with modulus δE, which is termed the persistent elastic modulus. In the case of positive loads (curve *3*), an increase in the sliding velocity w (a reduction in κ) leads to a reduction in the size of the contact area. This effect is similar to the effect of pivoting when sliding over a viscoelastic body when there is no adhesion [2]. In the case of negative values of the load, the dependence of the size of the contact area on the velocity is non-monotonic: at high velocities (low κ) there is a region in which the size of the contact area decreases when the sliding velocity decreases (curve *2*). The region expands when the absolute value of the negative load $\bar{P}$ increases. At fairly high absolute values of the load, a decrease in velocity leads to a reduction in the size of the contact area to zero, followed by separation of the interacting surfaces (curve *1*). Thus, the presence of adhesion leads, for negative loads, to the reverse effect to pivoting: when the velocity increases, the separated surfaces come into contact, and here the contact area increases as the velocity increases.

Curve *2'* on the left-hand side of Fig. 3 was obtained with the same parameter values as curve *2* but with a different viscosity $\delta = 10$. Curve *2'*, unlike curve *2*, has a monotonic form, i.e. an increase in the viscosity parameter δ led to a reduction in the effects related to adhesion.

The shift in the contact area $(b - a)/l$, graphs of which are given on the right-hand side of Fig. 3, is positive for positive loads, i.e. the contact area is shifted in the direction of motion of the cylinder (curve *3*), and here the shift reaches its maximum at a certain value of κ and approaches zero as $\kappa \to 0$ and $\kappa \to \infty$. Under negative loads the contact area is shifted in the opposite direction (the shift becomes negative – curves *1* and *2*).

The results indicate that the regions of contact and adhesion interaction are shifted with respect to the axis of symmetry of the cylinder, and the distribution of the contact pressure is also asymmetrical. This leads to the emergence of a tangential force acting on the cylinder from the viscoelastic half-space, despite the fact that the formulation of the problem assumes zero shear stresses at the boundary of the half-space ($\tau_{xy} = 0$ at $y = 0$).

We will calculate the tangential force T that must be applied to the cylinder in order to ensure that it moves at a constant velocity over the boundary of the viscoelastic half-space. This force is equal to the component of the force of reaction of the half-space along the Ox axis. Since the size of the region of interaction $a_1 + b_1$ is much smaller than the radius R of the cylinder, the following relation (Fig. 1) holds

$$T = \int_{-a_1}^{b_1} p(x)\sin\varphi(x)dx \approx \frac{1}{R}\int_{-a_1}^{b_1} xp(x)dx \tag{4.1}$$

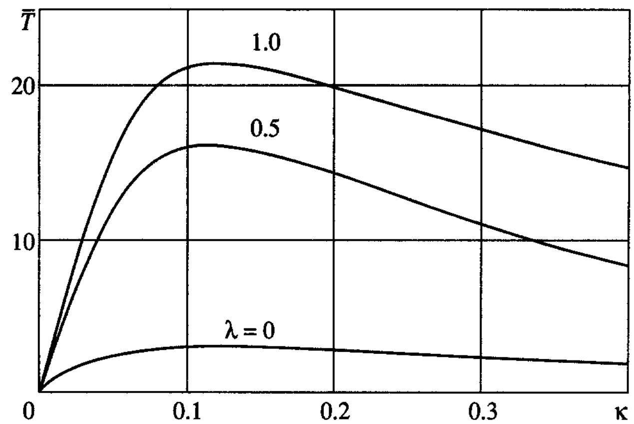

Fig. 4

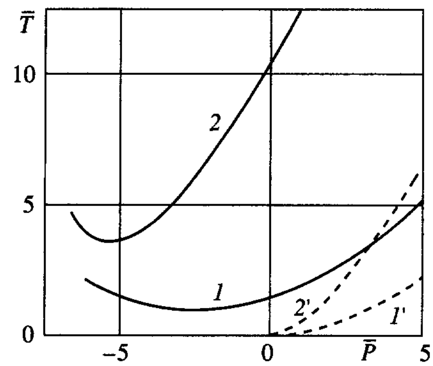

Fig. 5

Reducing equality (4.1) to dimensionless form, we obtain

$$\bar{T} = \frac{9\pi T}{2\gamma} = \int_{-a}^{b} \xi \bar{p}(\xi) d\xi = \int_{-a/l}^{b/l} \xi \bar{p}(\xi) d\xi - \frac{\lambda}{2l^2}(b_1^2 - b^2 + a^2 - a_1^2) \tag{4.2}$$

Figures 4 and 5 show graphs of $\bar{T}$, calculated by means of formula (4.2), as a function of parameters κ and $\bar{P}$. The curves in Fig. 4 were obtained for $\delta = 3$, $\bar{P} = 3$ and various values of the adhesion parameter λ. The results indicate that the tangential force is non-zero over a certain range of values of κ and approaches zero as $\kappa \to 0$ and $\kappa \to \infty$. These limiting cases, as pointed out, correspond to the solution of the problem in an elastic formulation. The case where $\lambda = 0$ corresponds to contact without adhesion. An increase in the adhesion parameter λ leads to an increase in the tangential force.

The results also indicate that the tangential force T acting on the cylinder from the half-space is always directed opposite to the motion of the cylinder, for both positive and negative values of the load. The dependence of this quantity on the load $\bar{P}$ is shown in Fig. 5 for cases of adhesion (curves *1* and *2*) and no adhesion (curves *1'* and *2'*). Curves *1* and *1'* correspond to $\kappa = 0.01$, and curve *2* and *2'* correspond to $\kappa = 0.1$. When there is no adhesion ($\lambda = 0$) the tangential force of resistance to motion of the cylinder is equal to zero when $\bar{P} = 0$ and increases monotonically when $\bar{P} > 0$. Allowance for adhesion leads to a non-monotonic dependence of T on the load $\bar{P}$ with a minimum in the region of negative loads.

5. CONCLUSIONS

Analysis of the solution of the problem of the sliding of a rigid cylinder over a viscoelastic half-space when there is adhesion enables us to draw the following conclusions.

1. For positive loads on the cylinder (pressing it against the half-space), the contact area is shifted with respect to the axis of symmetry of the cylinder in the direction of motion of the cylinder, and here

an increase in the sliding velocity leads to a reduction in the size of the contact area (the "pivoting effect"). These effects are similar to those obtained when adhesion is ignored.

2. For negative loads on the cylinder (separating it from the half-space) that are sufficiently large in absolute magnitude, the contact area is shifted in the opposite direction to the motion of the cylinder. The dependence of the size of the contact area on the velocity becomes non-monotonic. For a constant negative load, surfaces not in contact may come into contact when the velocity increases.

3. Allowance for adhesion leads to an increase in the tangential force of resistance to motion of the cylinder (the deformation component of the friction force). This force is directed against cylinder motion both under positive and negative loads. The resistance force depends non-monotonically on the applied load, having a maximum in the region of negative loads. An increase in the adhesion parameter λ leads to an increase in the resistance force.

4. An increase in the viscosity parameter δ leads to a reduction in the size of the contact area and to an increase in the displacement of this area with respect to the axis of symmetry of the cylinder, and also to an increase in the deformation component of the friction force. When $\delta = 1$, a solution is obtained that is identical with the solution of the problem for the contact of an elastic half-plane with a cylinder when there is adhesion.

This research was supported financially by a Grant of the President of the Russian Federation for Young Russian Scientists (MK-1345.2003.1) and the Russian Foundation for Basic Research (04-0100766).

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Translated by P.S.C.